The Actuarial Profession
making financial sense of the future

Compound Interest

Actuaries are professional mathematicians who look at the future. For example:

Problem
How much interest would you earn on £10,000 in an account with 0.4% p.a. for 8 years?
How much would this be after tax is deducted?
How has your investment grown in *real terms* (*ie* taking into account inflation)?

Method 1

We work out the interest each year and add it on.

To calculate compound interest on **£1200** invested for **3 years** at **5%** p.a.
5% = **× 0.05**
year 1: £1200 × 0.05 = £60 total at year end = £1200 + £60 = £1260
year 2: £1260 × 0.05 = £63 total at year end = £1260 + £63 = £1323
year 3: £1323 × 0.05 = £66.15 total at year end = £1323 + £66.15 = £1389.15
total amount or balance = **£1389.15**
interest = £1389.15 - £1200 = **£189.15**

1. Work out the **balance** on these investments:
a) £800 invested for 3 yrs at 5% p.a.
b) £1600 for 4 yrs at 10% p.a.
c) £1500 invested for 3 yrs at 12.5% p.a
d) £3000 invested for 2 yrs at 3.9% p.a

2. Work out the **interest** on:
a) £400 invested for 2 yrs at 6% p.a.
b) £1000 invested for 5 yrs at 3% p.a.

Method 2

We use **percentage multipliers** to get the new total for each year in one go.

To calculate compound interest on **£1200** for **3 years** at **5%** p.a.
100% start + 5% interest = 105% so **× 1.05**
year 1: total at year end = £1200 × 1.05 = £1260
year 2: total at year end = £1260 × 1.05 = £1323
year 3: total at year end = £1323 × 1.05 = £1389.15
total amount = **£1389.15**
interest = £1389.15 - £1200 = **£189.15**

3. What would the **percentage multiplier** be for each of these?
a) 7% interest
b) 12% interest
c) 3% interest
d) 17.5% interest
e) 1.5% interest
f) 0.4% interest

4. Calculate the **balance** for each of these investments using percentage multipliers:
a) £600 invested for 2 yrs at 8% p.a.
b) £1800 invested for 3 yrs at 15% p.a.
c) £1200 invested at 2.5% p.a. for 4 yrs
d) £5000 invested at 1.4% p.a. for 3 yrs

5. Work out the **interest** earned on these investments:
a) £500 for 3 yrs at 7% p.a.
b) £2000 for 4 yrs at 2.6%

Method 3

We use powers with our percentage multipliers to do all the years in one go.

To calculate compound interest on **£1200** for **3 years** at **5%** p.a.
For three years, we need to multiply by 1.05 three times:
£1200 × 1.05 × 1.05 × 1.05 = £1389.15

using powers this can be done in one step:
total amount = £1200 **× 1.05^3** = **£1389.15**
interest = £1389.15 - £1200 = **£189.15**

NB Remember the power key on your calculator is $\boxed{x^y}$ on a Casio, $\boxed{y^x}$ on a Sharp.

6. Use **powers** to work out the **balances** of these investments in one go:
a) £700 invested for 3 yrs at 2% p.a.
b) £1200 invested for 5 yrs at 12% p.a.
c) £8000 invested at 7.5% p.a. for 4 yrs
d) £2400 invested at 3.25% p.a. for 6 yrs

7. Now work out the **interest** earned on these investments:
a) £300 invested for 7 yrs at 9% p.a.
b) £3600 invested for 5 yrs at 2.6%

Income Tax

Unfortunately, the government charges 20% tax on the interest you earn on your account.
So you will only get 80% of the interest earned.
eg on 3% interest p.a. you will only get 80% of 3% = 0.80 × 3% = 2.4% p.a.

8. What are the interest rates after tax (called the **net interest** rates) on these bank accounts?
a) 7% interest p.a.
b) 4% interest p.a.
c) 2.5% interest p.a.

9. Work out the **net interest** earned on these investments:
a) £900 for 5 yrs at 8% p.a.
b) £1500 at 3.5% p.a. for 6 yrs

Inflation
Sometimes items that you buy in a shop become more expensive every year.
The amount they go up by is called **inflation**, which is currently about **2.4% p.a.**
eg CDs currently cost about £12.99, so in 3 years they could cost £12.99 × 1.024^3 = £13.95

10.
a) How many CDs could you buy with £100 **now**?
b) A current account pays 0.1% p.a. interest.
A student (who doesn't pay tax) invests £100 in this bank account for 3 years.
How much money will she have in 3 years time?
c) How many CD's (at their new price) will she be able to buy in 3 years time?
d) What do you notice?

Challenges:

11 James invests £100 in a current account which pays 4.07% p.a. James pays tax.
a) What will his investment be after 5 years?
b) How many CD's will he be able to purchase in 5 years time?
c) By what percentage has his investment grown by in *real terms*?

12. Jasneet invests £500 in an account that pays 3% p.a. She doesn't pay tax.
How long will it take for her to double her money?

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Teacher's Notes

Use of these materials

These materials are designed for students studying the new **Higher GCSE Course**.

They could be used when studying compound percentages (Ma 2 xxx) or as extension material for the more able student or as an alternative 'real-life' lesson in the course.

It is not expected that all students will finish the sheet.
The sections on income tax and inflation and the challenges are meant to really push the most able and keep them thinking whilst others are completing the basic material.
The steady progression means that more able students could start later on.

Lesson Plan

Introduction

Explain what **actuaries** are and the kinds of things they do with maths.

> **Actuaries** are professional mathematicians who look at the future. Traditionally, actuaries have worked for insurance companies or pension funds, calculating the premiums charged by using probability models of mortality and other events. Now they are being employed more widely to assess the long-term financial implications of business decisions.

Outline the **initial problem** (perhaps writing it on the board or displaying it on an OHP).
Discuss why **interest** is paid.
Discuss why it is called **compound interest** and what **p.a.** stands for.

> Banks or building societies use your money to invest in the stock market. They then pay **interest** to you for loaning them your money. Accounts which guarantee them the money for longer periods pay more interest (as they have longer to play with it on the stock market to make money), whereas a current account pays less interest (as you could withdraw it at any time).
>
> **Compound interest** means that the interest you earn is added to your account. You can then earn interest on your interest (*ie* it is compounded). This will mean that there will be more interest next year since there is now more money in the bank (assuming you don't take any money out!)

Method 1

Carry out an **example** using students' suggestions for the investment and interest for **1 year**.
Get them to explain how *they* would calculate the interest and then the total amount.
Summarise the method used for the rest of the class.

Ask the students what the **interest** will be for the **second year**.
Take a selection of answers and get students to **vote** on which answer they think is correct.

> **Common Misconception**
> Some students will think that the interest for the 2nd year will be the same as the 1st.

Explain that this is not the case as it is, for example, **<u>not</u> 5% of the *original* amount** but 5% of the new balance at the end of one year. Since it is 5% of more money the interest will be more.
Now get the students to calculate the **amount** at the end of the **second year**. Check the answers.
Then get them to work out the **amount** at the end of the **third year**. Check the answers.
Ask students what **interest** has been paid in total **over the three years**.
Get them to explain how they worked this out.

Carry out a **second example** using student's suggestions for money and interest.

For a top set move onto an explanation of Method 2 straight away.
Otherwise start the students on Q1 now, going round and checking their work.
Stop the class when the first few have reached method 2 (and hopefully understood it from the sheet).
Mark the rest of the questions and proceed with the explanation below.

Method 2

Point out the long-winded nature of the first method and how we could do with a quicker way.
Explain we **start** with all our money (*ie* **100%**) and so when we **add** on the **interest** of, say, **5%** we will have **105%** ***in total***. So to work out the new total in one step we need to **×1.05**.

Get students to give the total percentage (and the corresponding multiplier) for other accounts with 8% interest, 12% interest, 1% interest, 1.5% interest, 3.6% interest and 0.6% interest.

Now carry out an **example** using this new method to get the total at the end of each year.
Do the first year together. Get students to do the 2nd and 3rd years and check them together.

Do another example if needed or move onto Q1, 2 or 3 for top sets or Method 3 otherwise.
Display or regularly read out answers as students progress through the questions.
Stop the students after the first few reach method 3 and briefly explain it to the class.
Students should then continue from where they left off.

Method 3

To work out amount on £1200 at 5% for 3 years, you would do:
£1200 ×1.05 = … ×1.05 = … ×1.05 = ….
ie **£1200 × 1.05 × 1.05 × 1.05**
Ask students what the shorter way is of writing 1.05 × 1.05 × 1.05
Explain that they need to work out **£1200 × 1.05^3** to do it in one go.

Remind students how to do this on their calculators (*ie* 1200 × 1.05 [x^y] 3 on a Casio).
Get students to give the shortcut way of working out other examples.

Summary

Summarise that students have learnt how to work out the value of future investments like actuaries do. Get some students to explain the method they have learned. Now show how they can solve the problem posed at the beginning of the lesson or get them to do it.

Since most students might not have got onto the tax and inflation, briefly summarise the issues:

The Government charges 20% tax on interest earned (although the students shouldn't be paying tax and so they should've filled out a form with their bank to ensure this is the case). So the interest rates quoted are normally the **gross interest** (*ie* before tax) and the **net interest** is the amount after tax. For example on an account with 5% p.a. you will only get 80% of this *ie* 80% of 5% = 4%.

Inflation is how much more expensive things are becoming each year. This is currently 2.4% and so if students put money into an account with 1% interest they are effectively losing money as they get 1% more money but things are costing 2.4% more!

Students wishing to find out more about a career as an Actuary should visit www.actuaries.org.uk

Compound Interest

Answers

1. a) totals: *800*, 840, 882, **£926.10**
b) totals: *1600*, 1760, 1936, 2129.60, **£2342.56**
c) totals: *1500*, 1687.50, **£1898.44** (nearest penny)
d) totals: *3000*, 3117, **£3238.56**

2. a) totals: *400*, 424, £449.44 interest = **£49.44**
b) totals: *1000*, 1030, 1060.90, 1092.73, 1125.51, £1159.28 interest = **£159.28**

3. a) 1.07 b) 1.12 c) 1.03 d) 1.175 e) 1.015 f) 1.004

4. a) *600*, 648, **£699.84** b) *1800*, 2070, 2480.50, **£2737.58**
c) *1200*, 1230, 1260.75, 1292.27, **£1324.58** d) *5000*, 5070, 5140.98, **£5212.95**

5. a) *500*, 535, 572.45, £612.52 interest = £612.52 - £500 = **£112.52**
b) *2000*, 2052, 2105.35, 2160.09, £2216.25 interest = £2216.25 - £2000 = **£216.25**

6. a) 700×1.02^3 = **£742.85** b) 1200×1.12^5 = **£2114.81**
c) 8000×1.075^4 = **£10683.75** d) 2400×1.0325^6 = **£2907.71**

7. a) 300×1.09^7 = **£548.41** b) 3600×1.026^5 = **£4092.98**

8. a) 0.80 × 7% = **5.6%** b) 0.80 × 4% = **3.2%** c) 0.80 × 2.5% = **2%**

9. a) 0.80 × 8% = 6.4% 900×1.064^5 = £1227.30 interest = £1227.30-£900 = **£327.30**
b) 0.80 × 3.5% = 2.8% 1500×1.028^6 = £1770.31 interest = £1770.31-£1500 = **£270.31**

10. a) £100 ÷ £12.99 = **7.7 CD's** b) 100×1.001^3 = **£100.30**
c) £100.30 ÷ £13.95 = **7.2 CD's**
d) Items are becoming 2.4% more expensive each year and her money is only growing at 0.1% each year. Therefore**, she is losing money in *real terms*** and will be able to buy less.

11. a) 0.80 × 4.07% = 3.256% 100×1.03256^5 = **£117.38**
b) In 5 years, CD's will cost £12.99 × 1.024^5 = £14.63
So James will be able to buy £117.38 ÷ £14.63 = 8.0 CD's.
c) Originally, James could buy 7.7 CD's, in 5 years he can buy 8.0 CD's.
The percentage growth in real terms is 0.3 ÷ 7.7 = 3.9% in 5 years.

12. Between 23 and 24 years.
viz. £500×1.03^{23} = £986.79 and £500×1.03^{24} = £1016.40
Incidentally, she will become a millionaire in about 257 years.

Answer to Original Problem

a) £10,000 × 1.004^8 = £10324.52 interest = **£324.52**
b) 0.80×0.4% = 0.32% £10,000 × 1.0032^8 = £10258.89 interest = **£258.89**
c) Can buy £10,000 ÷ £12.99 = 769.8 CD's now.
CD's will cost £12.99 × 1.024^8 = £15.70
They will be able to buy £10258.89 ÷ £15.70 = 653.4 CD's in 8 years time.
This is a **loss** of 116.4 CD's, which is 116.4 ÷ 769.8 = **15.1% over 8 years**

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